Pythagorean Theorem on Ice

By Mark Levi

In the spirit of winter fun, here is a short "skater's proof" of the Pythagorean theorem. Starting in the corner O of a skating rink with the walls along the x and y axes, wearing perfectly slippery shoes on the perfectly slippery ice, I push away from the y-wall, acquiring speed a in the x-direction (see Figure 1). Next, I push away from the x-wall, gaining speed b in the y-direction. With the first push I acquired kinetic energy $ma^2/2$, and with the second push I added $mb^2/2$ to my kinetic energy. Indeed, the fact that I was sliding along the x-wall is irrelevant, because my gloves are perfectly slippery; it feels the same as if the wall were not sliding by at all, just like during the first push.[1] After the two pushes, my speed c is the hypotenuse of the velocity triangle, and thus my kinetic energy is $mc^2/2$. But this energy is the accumulation of the two previous contributions:

$$\frac{mc^2}{2} = \frac{ma^2}{2} + \frac{mb^2}{2},$$

implying $a^2 + b^2 = c^2$. All this can be summarized by saying that the energies add as scalars, while the velocities add as vectors. Of course, the above is not meant as a rigorous proof and is rather an interpretation, or a physical incarnation, of the Pythagorean theorem.

MATHEMATICAL CURIOSITIES
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The figure in this article was provided by the author.

Mark Levi (levi@math.psu.edu) is a professor of mathematics at the Pennsylvania State University.

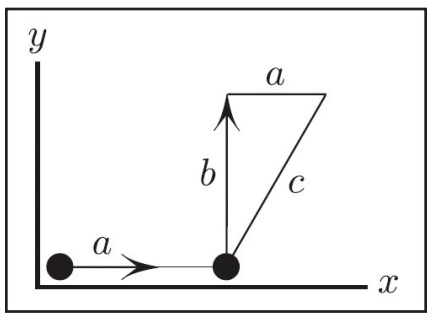

Figure 1. *A kinetic energy "proof" of the Pythagorean theorem.*

[1] it is here that the orthogonality of the walls is used.